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# US ARMY ELECTRONICS RESEARCH & DEVELOPMENT ACTIVITY

DME DATA REDUCTION

BY

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USA ERDA-1

JANUARY 1963

WHITE SANDS MISSILE RANGE
NEW MEXICO

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INSTRUMENTATION DEPARTMENT
U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT ACTIVITY
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#### FOREWORD

The basic outputs of the Distance Measuring Equipment (DME) are the digitized phase delays of four so-called data frequencies. In its initial configuration, the DME contained a small special-purpose digital computer which was to derive a range word from the four phase words. It was soon realized that the fixed-wired program was far inferior to a flexible program which could cope with many situations in which the fixed program would fail. In the absence of any other information, all four phasemeter outputs are required for computation of a nonambiguous range word, but there are many cases where ambiguity resolution can be accomplished on the basis of prior DME data or even data of other systems. While, with the fixed program, data is lost when any one of the four phasemeters fails, a flexible program can continue operating with any combination of one, two, or three phasemeters. In addition, flexible programs may provide higher precision by averaging over the four phasemeter outputs.

All the benefits of flexible programming can be obtained only if the range-word synthesis from phase words is fully understood. Therefore the major part of this report is devoted to describing this range-word synthesis in detail. While the actual DME is based on binary relationships to provide data ideally suited for digital computers, an equivalent decimal system is discussed first in order to show the various computational steps more clearly. At the end, two programs for the binary DME are developed and a description of the DME tapes is included.

A remark pertaining to the symbolism employed in this report seems appropriate. The actual, correct phase delays associated with distances are denoted by small p's. These p's are principally unavailable; when they appear in equations, they are to show or prove the mathematical consistence. Conversely, phasemeter outputs and all digital quantities derived from them in the course of the program are denoted by P's or other capital letters, respectively. While a capital letter stands for a binary number with a definite numerical range and bit weighting, the small p's may stand for either the total phase delay or that part of the total phase delay dealt within the particular equation.

This report is intended to be the first step to devising intelligent DME data reduction routines. It is hoped that it provides the background for the development of more advanced methods.

#### **ABSTRACT**

The ambiguity-resolution problem for a Distance Measuring Equipment (DME), using four independent modulation frequencies in the 500-kc region, is described in mathematical terms. The method is developed for a DME model with decimal modulation-frequency relationships and then applied to the actual system whose modulation frequencies are related by binary numbers. Finally, two digital-computer routines providing for both ambiguity resolution and accuracy increase by averaging over the four phase measurements are developed.

#### CONTENTS

	PAGE
FOREWORD	iii
ABSTRACT	· <b>v</b>
GENERAL	1
EQUIVALENT DECIMAL DMS	1
AMBIGUITY RESOLUTION WITH PERFECTLY ACCURATE PHASEMETER OUTPUTS	4
FINE-AMBIGUITY RESOLUTION WITH PROVISION FOR PHASEMETER ERRORS	7
COARSE-AMBIGUITY RESOLUTION WITH PROVISION FOR PHASEMETER ERRORS -	11
VERY-COARSE-AMBIGUITY RESOLUTION WITH PROVISION FOR PHASEMETER ERRORS	13
ACCURACY DIPROVEMENT BY AVERAGING	15
AN IMPROVED METHOD OF AMBIGUITY RESOLUTION	16
THE BINARY DNG	19
THE IMPROVED METHOD OF AMELGUITY RESOLUTION IN BINARY FORM	22
SMOOTHING	24
THE TAPE WIRMAT	2h

#### DME DATA REDUCTION

#### 1. GENERAL

In principle, DME range is determined as follows: four "ranging frequencies" are modulated on a UHF carrier, transmitted to the airborne transponder, detected, remodulated on a new UHF carrier, retransmitted to the ground station and demodulated (Fig. 1). The phase delay of the demodulated ranging frequencies yields the range to the target. 360 degrees of phase delay in the loop ground-target ground correspond to the following one way distances:

Rang	ing Frequencie	8	<u>O1</u>	<u>ne-Way Distan</u>	ces	(1-1)	
Very Fine	= VF = f1	480.234 kc	2 <sup>10</sup> ft =	1,024ft =	0.19	miles	
Fine	$= F_n = 2^{-4}f1$	30.015 kc	2 <sup>14</sup> ft =	16,384ft =	3.1	miles	
Coarse	$= Cs = 2^{-8}f1$	1.875 kc	2 <sup>18</sup> ft =	262,144ft =	49.6	miles	
Very Coarse	$= vc = 2^{-12}f1$	0.117 kc	$2^{22}ft = 4$	4,194,304ft =	795	miles	

Range information is obtained by combining the data obtained from the four phase meters. If phase can be measured accurate to one degree, 1024/360 = 2.9 feet range accuracy is obtained. With the given relation of 1:16 between ranging frequencies, the range data of the lower ranging frequencies can be corrected to conform with the range data of the next higher frequencies, as long as the error in the lower (coarser) frequency is less than  $\pm \frac{1}{2}360/16 = \pm 12^{\circ}$ . This process is called ambiguity resolution and is in detail described in the following. The binary relationships of the ranging frequencies make the data particularly suitable for digital computers. For reasons of spectrum conservation and carrier frequency isolation, the following "data frequencies" (being sum and differences of the ranging frequencies) are modulated on the transmitter:

$$f1 = VF$$
 = 480.234 Kc (1-2)  
 $f2 = VF-Fn$  = 480.234-30.015 = 450.219 Kc  
 $f3 = VF+Cs$  = 480.234+1.875 = 482.109 Kc  
 $f4 = FV-Cs-VC$  = 480.234-1.875-0.117 = 478.241 Kc

#### 2. EQUIVALENT DECIMAL DME

In order to spare at this stage the cumbersome notation of a DME with binary-related ranging frequencies we described first an equivalent system with decimal-related frequencies. Thus we choose the basic data

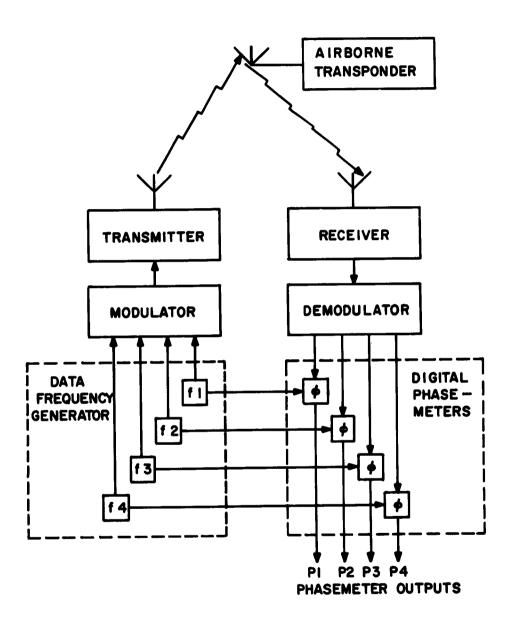


Figure 1 PRINCIPLE OF DME

frequency, f1 = VF, such that  $360^{\circ}$  loop-range delay ( $180^{\circ}$  one-way delay) corresponds to exactly 1000 feet. Phase angle is measured digitally, with 1000 bits corresponding to  $360^{\circ}$  loop-range delay. Thus one bit equals one foot one-way distance. In this decimal system, the ranging frequencies become:

VF = f1; 
$$360^{\circ}$$
 loop delay = 1,000ft one-way distance (2-1)  
Fn =  $\frac{1}{10}$  f1;  $360^{\circ}$  loop delay = 10,000ft one-way distance  
Cs =  $\frac{1}{100}$  f1;  $360^{\circ}$  loop delay = 100,000ft one-way distance  
VC =  $\frac{1}{1000}$  f1;  $360^{\circ}$  loop delay = 1,000,000ft one-way distance

The actual data frequencies used would be:

f1 = VF (2-2)  
f2 = 
$$\frac{9}{10}$$
f1 = VF-Fn  
f3 =  $\frac{101}{100}$  f1 = VF+Cs  
f4 =  $\frac{100-10-1}{1000}$ f1 =  $\frac{989}{1000}$  f1 = VF-Cs-VC

(The scheme would call for  $f4 = \frac{999}{1000}$  fl, but separating f4 and fl would require an extremely sharp filter with undesirable phase characteristics). With all four frequencies derived from a common crystal oscillator, the above frequency relationships are exactly satisfied at all times.

Since phase delay is proportional to frequency, the digital phases corresponding to an actual distance of 1000 feet are as follows:

Phase repeats itself when exceeding 360 degrees; therefore phase delays larger than 360 degrees cannot be non-ambiguously measured. The same is true for the digital numbers obtained from the DME phasemeters. While the total phase delays, denoted by small p's, usually amount to several hundred wavelengths, the phasemeters only provide that fraction of a wavelength by which an unknown integral number of wavelengths is exceeded. The DME phasemeters do not accumulate phase, as is done in DOVAP digitizers. In the decimal DME presently under consideration, the total phase delays p are six-digit numbers, while the phasemeter readings P contain only the last three digits of the total phase delay p.

#### AMBIGUITY RESOLUTION WITH PERFECTLY ACCURATE PHASEMETER OUTPUTS

We consider first the ambiguity resolution process for the case in which all phasemeters read absolutely accurate. As an example, we consider the determination of a range of 756,999.000 feet.

$$p1 = p1$$
  $p1 = 756,999.000$  (3-1)

$$p1 = p1 p1 = 756,999.000 (3-1)$$

$$p2 = \frac{9}{10} p1 756,999.000 (3-2)$$

$$p2 = \frac{75,699.900}{681,299.100}$$

$$p3 = \frac{101}{100} p1 \qquad 756,999.000 \qquad (3-3)$$

$$p3 = \frac{7,569.990}{764,568.990}$$

The above total phase delays are exact mathematical relationships with respect to pl. The digitized phasemeter reading include neither the first three digits, which correspond to multiples of whole wavelengths, nor the digits following the decimal point, which are beyond the resolution of the digital system. Numbering the digits by their power of ten, the phasemeters only provide digits 2, 1, 0 of the total phase delay. To avoid round-off errors, we carry the (unavailable) digits following the decimal point along by the following notation:

$$p1 = P1$$
  $p1 = p1$  (3-5)  
 $p2 = P2+Q2$   $p2 = p2-Q2$   
 $p3 = P3+Q3$   $p3 = p3-Q3$   
 $p4 = P4+Q4$   $P4 = p4-Q4$ 

In these equations, P1 is assumed accurate, and the Q's are smaller than one bit. Also, these equations ignore the first three digits of the p's. In our example:

The problem is to resolve the ambiguous phase information, i.e. compute a nonambiguous range from the four phasemeter readings. The range ambiguities are resolved in three consecutive steps.

Under the assumption that all readings are accurate, P1 = 999 implies that digits 0, 1, 2 of the range are 999. This portion of the range we call S1. By itself, S1 is ambiguous because the actual range may be 999; 1,999; 2,999;...; 998,999; 999,999 feet. In fact, there are exactly one-thousand possibilities differing from the correct range by multiples of 1000 feet. These possible values are called fine ambiguities because they could be resolved by a phase measurement of the fine ranging frequency (Fn). Nine hundred of the thousand fine ambiguities are eliminated in the first step:

$$\frac{1}{10} \text{ s2 = p1-p2 = p1-} \frac{9}{10} \text{ p1 = } \frac{1}{10} \text{ p1}$$

$$\text{s2 = p1}$$
(3-7)

This equation appears as a mere identity when operating with the exact p's. However, when operating with the three-digit numbers

$$\frac{1}{10}$$
 S2 = P1-P2 = p1-(p2-Q2) = p1-( $\frac{9}{10}$  p1-Q2) =  $\frac{1}{10}$  p1+Q2 (3-8)

$$S2 = p1+10Q2$$
 (3-9)

$$S2C = S2-10Q2 = p1$$
 (3-10)

the difference  $\frac{1}{10}$  S2 is again a three-digit number so that S2 becomes a four-digit number whose last digit is zero. If such a four-digit number is to equal p1, we infer that it equals digits 3, 2, 1 of p1. In numbers,

$$\begin{array}{ccc}
P1 & 999 \\
P2 & 299 \\
\hline
\hline
10 & S2 & 700
\end{array}$$
(3-11)

$$S2 = 7000 \text{ ft}$$
 (=Fn/U)  
 $S2C = 7000 - 1 = 6999 \text{ ft}$  (=Fn/C)

we see first that S2 differs by one bit from the digits 3, 2, 1, 0 of p1. This difference is not an equipment error, but a consequence of the quantity Q2 which is below the resolution of the phasemeters. The small zero in S2 is an insignificant digit which is always zero due to the multiplication by ten. The correct value for S2, which we call S2C, is to equal P1 in the last three digits and is hence S2C = 6999. The technique of obtaining S2C from S2 will be described when discussing ambiguity resolution with non-ideal phasemeters (Section 4).

The DME manufacturer, Cubic Corporation, denotes the quantities S2 and S2C by Fn/U and Fn/C, which stands for "fine uncorrected" and "fine corrected". These symbols refer to the ranging frequencies defined in Equations 3: We would have measured a nonambigious phase Fn/C, had

we actually used the "fine" ranging frequency. Since sums and differences of the ranging frequencies are used rather than the ranging frequencies themselves, Cubic's notation appears little helpful for someone who has no experience with Cubic's previous DME system which did actually use the ranging frequencies. Thesefore, we will use S-symbols for distance-like quantities, but we will mention Cubic's notation to provide a background for understanding Cubic's literature. In the first step we obtained digit 3 of pl, or the range R, respectively. We have just removed nine hundred of the possible one-thousand values for the range R, leaving 6,999; 16,999; ...986,999 996,999 feet. From these one-hundred possibilities, called coarse ambiguities because they appear as multiples of 10,000 feet, ninety are removed by the second step:

$$\frac{1}{100}$$
 S3 = P3-P1 = (p3-Q3)-p1 = ( $\frac{101}{100}$  p1-Q3)-p1 =  $\frac{1}{100}$  p1-Q3 (3-12)

$$S3 = p1-100Q3$$
 (3-13)

$$S3C = S3+100Q3 = p1$$
 (3-14)

Again  $\frac{1}{100}$  S3 is a three-digit quantity so that S3 becomes a five-digit quantity with two insignificant zeros at the end. The first digit of S3 is to provide digit 4 of the range R. In our example,

P3 568
P1 999-

$$\frac{1}{100}$$
 S3 569

S3 = 56900 ft (=Cs/U)

S3C = 56900+99 = 56999 ft (=Cs/C)

At this stage there are only ten possibilities left, namely 56,999, 156,999;...; 856,999; 956,999. These are called very-coarse ambiguities because they appear in multiples of 100,000 feet. The last step singles out the actual range by

$$\frac{1}{1000} S4 = P1-P4 - \frac{1}{100}S3$$

$$= p1 - (p4-Q4) - \frac{1}{100}(p1-100Q3)$$

$$= p1 - (\frac{989}{1000}p1-Q4) - \frac{1}{100}p1+Q3$$

$$= \frac{1}{1000}p1+Q4+Q3$$
(3-16)

$$S4 = p1+1000(Q4+Q3)$$
 (=VC/U) (3-17)

$$S4C = S4-1000(Q4+Q3) = R1 (=VC/C)$$
 (3-18)

In our example:

P1 999  
P4 672-  

$$\frac{1}{100}$$
 S3 569-  
 $\frac{1}{1000}$  S4 758

$$S4C = 756,999 \text{ feet } = R = (VC/C)$$

Thus we have singled out the actual range R in the following sequence:

Note however, that we did not show how to obtain the Q's. Thus the use of equations (3-10), (3-14), and (3-18) is premature at this stage.

#### 4. FINE-AMBIGUITY RESOLUTION WITH PROVISION FOR PHASEMETER ERRORS

While the previous section dealt with ideal phasemeters, for which the errors Q were below the resolution, this section discusses ambiguity resolution with phasemeters having outputs with errors larger than their resolution.

The fine-ambiguity resolution problem is graphically displayed in Figure 2. Three of the one-thousand fine ambiguities namely 5,999; 6,999; 7,999 feet are shown. If S2 comes close to 6,999 feet, for instance 6800 or 7000 feet, as shown as Cases 3 and 4, then 6,999 is clearly established as the correct value for S2, called S2C (correct).

In general, the correct value of the one thousand possible ones is likely to be the one to which S2 comes closest. To establish S2C = 6,999 on this basis, S2 would need to lie between 6,500 (499 feet low) and 7,490 (491 feet high), as indicated as Cases 1 and 2. Since S2 = 10(P1-P2) and P1 is considered an accurate reference at this time, P2 may be inconsistent with regard to P1 by  $\pm$ 49 units. Writing P2 = p2-Q2, as we did before, Q2 may now be a substantial error ranging from  $\pm$ 49 units to which fractional quantities like those in Equation 3-6 may be added. Note, however, that the likelihood of selecting a wrong S2C increases with

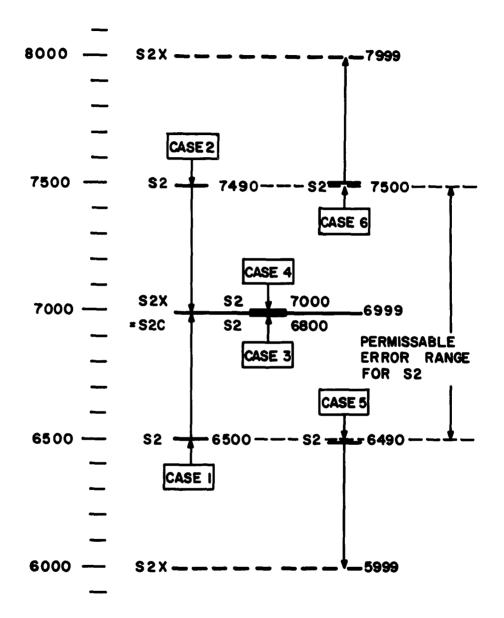


Figure 2 RESOLUTION OF FINE AMBIGUITIES

increasing Q2 and approaches 50% when Q2 approaches ±49. The arrows in Figure 2 indicate corrections by which S2 is made to conform with S2C. If S2 differs from S2C by more than ±500 feet, as displayed in Figure 2 as Cases 5 and 6, then a so-called ambiguity error is introduced in that the system selects a wrong S2C differing from the correct one by multiples of 1000 feet.

We consider now Cases 1 and 2 of Figure 2.

Case I	Case 2
S2 489 ft low	S2 491 ft high
P2 49 units high	P2 49 units low
Q2 =-49 plus a fraction	Q2 = 49 plus a fraction
P1 = 999	P1 = 999
P2 = 299+49 = 348	P2 = 299-49 = 250

First, we obtain S2 from (3-7)

Conn 1

Second, we establish the error Q2 according (3-10) as:

$$1002 = S2 - S2C$$
 (4-2)

Case 2

However, at this point, S2C is not known. We know only that the last three digits (digits 2, 1, 0) of S2C s.e to equal P1 (or S1). We express our ignorance of digit 3 by writing S2X instead of S2C.

$$10Q2X = S2-S2X$$
 (4-3)

Numerically, for the Cases 1 and 2, this amounts to:

Note that no mathematical operations are performed with an X-digit.

In Case 2 the possible errors (see Figure 3) are 491; 1491;... if S2C was lower than S2, and -509; -1509;... if S2C was above S2. Since we presupposed that the S2C is the S2X coming closest to S2, we need to look for the minimum 10Q2X, which is clearly 10Q2 = 491. The error term Q2 is opposite to the required correction of S2.

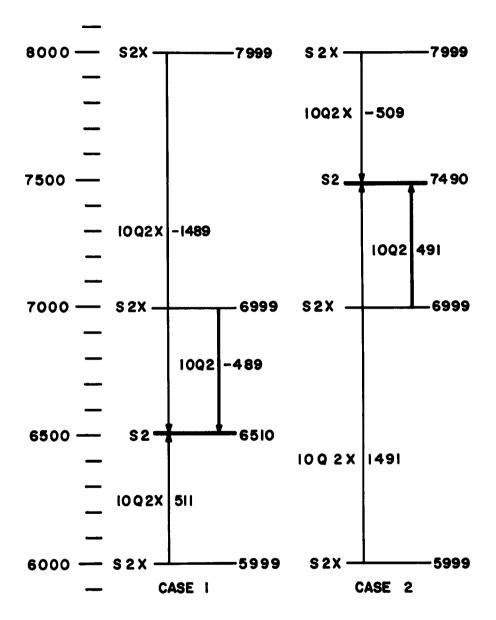


Figure 3 DETERMINATION OF ERROR Q2

In Case 1 the possible errors are 511; 1511;... if S2C was below S2, and -489; -1489;... if S2C was above S2. Here the minimum 10Q2X is 10Q2 = -489.

Note from the examples that 10Q2 is simply determined by:

$$10Q2 = 10Q2X$$
  $000 \le 10Q2X \le 499$  (4-5)  
 $10Q2 = - \text{complement } 10Q2X \le 501 \le 10Q2X \le 999$ 

If we arbitrarily define

then

10Q2 = 10Q2X if first digit is 0 to 4 10Q2 = minus 1000 complement of 10Q2X if first digit is 5 to 9.

Having obtained 10Q2, we obtain S2C from equation (3-10)

$$S2C = S2-10Q2 = p1$$
 (4-7)

Which amounts in the given examples to

Note that Q2 does not only determine the error in S2, but also the inconsistency of P2 with respect to P1. If we assume P1 and P2 equally good measurements, we can increase the accuracy by forming an average. However, at this point we assume P1 as an accurate reference, which we will correct after all ambiguities are resolved.

#### 5. COARSE-AMBIGUITY RESOLUTION WITH PROVISION FOR PHASEMETER ERRORS

After resolving the fine ambiguities, we have now one hundred possible range values left, all of them ending in S2C = 6,999. These possible range values we call S3X. Three of these possible values, namely S3X = 46,999; 56,999; 66,999 are shown in Figure 4. To establish S3C = 56,999 feet, S3 should not differ more than ±4900 feet from S3C. Let us again consider two cases:

Case 1 Case 2

S3 4900 feet low S3 4900 feet high
P3 49 units low P3 49 units high
Q3 = +49 plus a fraction Q3 = -49 plus a fraction

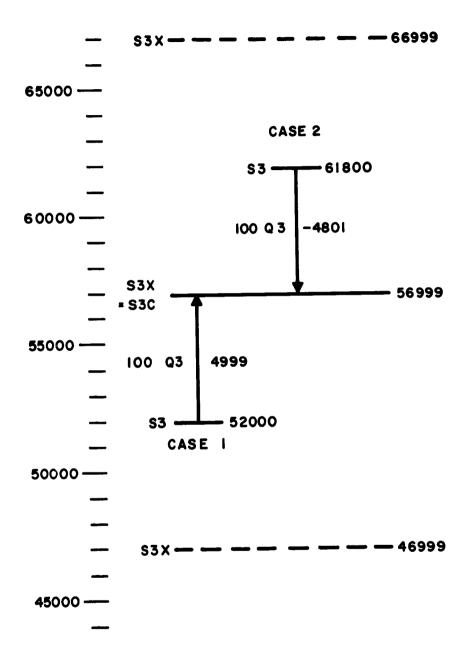


Figure 4 COARSE AMBIGUITY RESOLUTION

$$P1 = 999$$
  $P1 = 999$   $P3 = 568+49 = 617$ 

First, we obtain S3 from Equation 3-12

S3 52000

**S3** 

10003

Second, we establish Q3X according to Equation 3-14

$$100Q3X = S3X-S3$$
 (5-2)

S3 61800

where S3X = S2C augmented by an X as digit 4

and obtain 100Q3 by taking the negative 10,000 complement if the first digit of 100Q3X is 5 or more.

Finally we obtain S3C from Equation 3-14.

520oo

#### VERY-COARSE-AMBIGUITY RESOLUTION WITH PROVISION FOR PHASEMETER ERRORS

We are now left with ten possible range values, which are 56,999; 156,999;...; 856,999; 956,999 feet. The last five digits of all of them equal S3C. At this stage, a graphical representation of S4 is not as illuminating as the representations of S2 and S3, because S4 depends on P4 and S3, and therefore on both P3 and P4, as shown in Equations 3-12 and 3-16. In order to clearly demonstrate the method, we carry out as examples the four limiting cases of extreme Q3 and Q4. The extremes for Q3 are taken from the previous section.

Applying Equation 3-16

P1	999	999	999	999 (6-1)
P4	721-	<b>721</b> -	623-	623-
$\frac{1}{100}$ s3	<u>618-</u>	<u> 5<b>20-</b></u>	<u>618-</u>	<u>520-</u>
$\frac{1}{1000}$ S4	660	758	758	856
<b>S4</b>	660000	758000	758000	856000

Then, from 3-18, with S4X equaling S3C except for the unknown first digit,

$$1000(Q4X+Q3) = S4 - S4X$$
 (6-2)

from which we compute 1000Q4X by subtracting 1000Q3.

<b>S4</b>	660000	758000	758000	856000
S4X	X56999-	X56999-	X56999	X56999-
1000(Q4X+Q3)	X03001	X01001	X01001	X99001
1000Q3	-48010 <b>-</b>	49990-	-48010 <b>-</b>	49990-
1000Q4X	X51011	X51011	X49011	X49011

Note that the last subtraction is performed without carry into the X-digit. As was done before, the negative complement is taken when the first digit is 5 or more, hence

100004 -48989 -48989 49011 49011

We can now obtain the final range R from (3-18)

$$S4C = R = S4 - 1000Q4 - 1000Q3$$

<b>S</b> 4	660ooo	758000	758000	8 <b>56</b> 000	(6-3)
1000Q4	-48989-	-48989 <b>-</b>	49011-	49011-	
1000Q3_	-48010-	49990-	~48010 <del>~</del>	49990-	
S4C	756999	756999	756999	756999	

If we do not care to obtain S4 explicitly, we can combine the operations of (6-1) and (6-2) by using S3C instead of S3.

$$\frac{1}{1000} S40 = P1-P4-\frac{1}{100}S3C$$

$$= p1 -(\frac{989}{1000} p1-Q4) - \frac{1}{100}p1$$

$$= \frac{1}{1000} p1+Q4$$
(6-4)

Examples 1,2	Examples 3,4		
1000P1 999ooo	1000P1 999000		
1000P4 721000-	1000P4 623000-		
10S3C 56999o-	_10 S3C 56999o-		
S40 70801o	\$40 806 <b>0</b> 1o		
<u> </u>	S4X X56999-		
1000Q4X X51011	1000Q4X X49011		
1000Q4 -48989	1000Q4 49011		
S40 70801o	\$40 80601o		
1000Q4 -48989-	1000Q4 49011-		
SAC 756999	SAC 756999		

#### 7. ACCURACY IMPROVEMENT BY AVERAGING

For the purpose of ambiguity resolution we assumed P1 as an absolutely accurate reference and associated the Q-errors with the remaining P's. However, since the electronics for the four data frequencies is basically identical, the four phasemeter outputs can be expected to be equally precise. With the ambiguities resolved, the four phasemeter outputs are independent range measurements over which an average may be taken. The four indipendent range measurements are:

R1 = P1 = p1 (7-1)  
R2 = 
$$\frac{10}{9}$$
P2 =  $\frac{10}{9}$ (p2-Q2) = p1- $\frac{10}{9}$ Q2  
R3 =  $\frac{100}{101}$ P3 =  $\frac{100}{101}$ (p3-Q3) = p1- $\frac{100}{101}$ Q3  
R4 =  $\frac{1000}{989}$ P4 =  $\frac{1000}{989}$ (p4-Q4) = p1- $\frac{1000}{989}$ Q4

If we give the Q's equal weight in distance, the range average becomes

$$RA = R1 - \frac{1}{4} \left( \frac{10}{9} Q2 + \frac{100}{101} Q3 + \frac{1000}{989} Q4 \right)$$
 (7-2)

But the Q's may just as well be weighted equally in phase, then

$$RA = R1 - \frac{1}{4}(Q2 + Q3 + Q4)$$
 (7-3)

Different weighting factors may be chosen according to the actual noise level of each particular phasemeter.

#### 8. AN IMPROVED METHOD OF AMBIGUITY RESOLUTION

In Sections 4 to 6 we assumed an absolutely accurate Pl and found that the remaining P's may be inconsistent with regard to Pl by plus or minus 49 bits. The assumption of an accurate reference was made merely for mathematical convenience in the ambiguity-resolution process. In Section 7, after supposing that we had properly resolved the ambiguities, we admitted an error in Pl and corrected for it on the basis of an average over the four phasemeter readings.

An error in Pl reduces the 49-bit error margins of the other P's in one direction by the amount of the error in Pl. The ideal approach would use the correct average, rather than Pl, as the reference for ambiguity resolution. But the average is unavailable until the ambiguity resolution has been performed and is less likely to be correct the larger the error in Pl. There appear to be two practical approaches to obtaining the largest error margin for correct ambiguity resolution: one would perform averages after each ambiguity-resolution step, the other would check for correct resolution after the final range word has been obtained. In this report only the step-wise averaging method will be discussed.

We now assume pl and Pl to be correct only for the resolution of the fine ambiguities. Then we define

$$P2 = \frac{9}{10}p1-Q2 \tag{8-1}$$

and perform the fine-ambiguity resolution as described in Section 4. In this first step we will not gain anything over the method used in Section 3. Having obtained S3C, we average on the basis of equal phase accuracy of Pl and P2 to obtain a corrected Pl as

$$R2C = S2C - \frac{1}{2}Q2 = p1C$$
 (8-2)

For the resolution of the coarse ambiguities we assume plC and PlC (digits 2, 1, 0, -1 of R2C) to be correct. We define then

$$P3 = \frac{101}{100} p1C - U3 \tag{8-3}$$

and proceed as in Section 5, but using PIC instead of P1:

$$\frac{1}{100}S3A = P3-P1C = (\frac{101}{100}p1C-U3)-p1C$$
 (8-4)

$$S3A = p1C-100U3$$
 (8-5)

$$100U3X = S3X-S3A$$
 (8-6)

where S3X equals R2C augmented by an X in digit 5. With U3 obtained by complementing U3X if its first digit is 5 or more, we compute

$$S3C = S3A+100U3$$
 (8-7)

and perform another average

R3C = S3C-
$$\frac{1}{3}$$
U3 = p1CC (8-8)

taking into account (by the factor  $\frac{1}{3}$ ) that S3C is an average of two phasemeter readings.

In this way, we have obtained a doubly corrected P1 which we use as reference for the resolution of the very-course ambiguities. We now assume p1CC and P1CC (digits 2, 1, 0, -1,... of R3C) to be correct. We define then

$$P4 = p1CC-U4 \tag{8-9}$$

We follow now the routine of Section 6 but use PICC and R3C instead of P1 and S3C, respectively.

$$\frac{1}{1000}S4A = P1CC-P4 - \frac{1}{100}R3C$$

$$= p1CC - (\frac{989}{1000}p1CC-U4) - \frac{1}{100}p1CC$$
(8-10)

S4A = p1CC+1000U4

$$1000U4X = S4A - S4X (8-11)$$

where S4X equals R3C augmented by an X as digit 6. After obtaining U4 by the usual complementation,

$$S4C = S4A-1000U4$$
 (8-12)

we compute the final range RA as

$$R4C = S4C - \frac{1}{2}U4 = RA$$
 (8-13)

taking into account that S4C is an average of three phasemeter readings.

The routine described in this section increases the likelyhood for correct resolution of coarse and very-coarse ambiguities.

With correct ambiguity resolution, Equations 8-13 and 7-3 lead to nearly the same average range RA. Equation 8-13 can be written in the form:

RA = 
$$p1CC-\frac{1}{2}U4$$
 (8-14)  
=  $p1C-\frac{1}{3}U3-\frac{1}{2}U4$   
=  $p1-\frac{1}{2}Q2-\frac{1}{3}U3-\frac{1}{2}U4$ 

while Equation 7-3 can be expressed

$$RA = p1 - \frac{1}{2}Q2 - \frac{1}{2}Q3 - \frac{1}{2}Q4$$
 (8-15)

(8-20)

The U's can be expressed in terms of the Q's as follows:

U3 = 
$$(101/100)$$
plC-P3 =  $(101/100)$ (pl- $\frac{1}{2}$ Q2)-P3 (8-16)

$$U3+(101/100) \frac{1}{2}Q2 = (101/100) p1-P3 = Q3$$
 (8-17)

$$U3 = Q3 - (101/100) \frac{1}{2}Q2$$
 (8-18)

$$U4 = (989/1000) p1CC-P4 = (989/1000) (p1C-\frac{1}{3}U3)-P4$$

$$= (989/1000) (p1-\frac{1}{2}Q2-\frac{1}{3}U3)-P4$$

$$= (989/1000) \left[ p1-\frac{1}{2}Q2-\frac{1}{3}Q3+\frac{1}{3}(101/100)\frac{1}{2}Q2 \right]-P4$$

$$U4+(989/1000)\frac{1}{2} \left[ 1-\frac{1}{3}(101/100)Q2+(989/1000)\frac{1}{3}Q3 \right]$$

$$(8-20)$$

$$= \left[ (989/1000)_{P}1-P4 \right] = Q4$$

$$U4 = Q4-(989/1000)\frac{1}{3}Q3-(989/1000)\frac{1}{2}\left[ 1-(101/100)\frac{1}{3}\right]Q2 \qquad (8-21)$$

Using now 8-18 and 8-21 in Equation 8-14:

RA = 
$$p1-\frac{1}{3}\left[1-\frac{1}{3}(101/100)-\frac{1}{3}(989/1000)(1-(101/100)\frac{1}{3}\right]$$
 (8-22)  

$$-\frac{1}{3}\left[1-\frac{1}{3}(989/1000)\right]$$
 Q3-\frac{1}{3}\left[1-\left(811/600,000)\left] \right] \right] \right] \right] \right] \right] \right] \right] 2 \right] \frac{1}{3}\left[1-\left(811/600,000)\left] \right] \

where the last line very nearly equals Equation 8-15.

#### 9. THE BINARY DME

After discussing a comparatively simpler decimal system, we now return to the actual system, as was described in Section 1. This system differs from the decimal system in two ways: The ranging frequencies are related by 1:16 (as compared to 1:10) and  $360^{\circ}$  loop-range delay is digitized in  $2^{13}$  = 8,192 bits. The very-fine data frequency, fl, is chosen such that  $360^{\circ}$  loop-range delay equal  $2^{10}$  or 1024 feet. Thus one bit of Pl equals  $\frac{1}{8}$  foot. As convenient reference, a table of powers of 2 is given below:

2 <sup>21</sup>	2,097,152	210	1,024
2 <sup>20</sup>	1,048,576	29	512
2 <sup>19</sup>	524,288	28	256
2 <sup>18</sup>	262,144	27	128
2 <sup>17</sup>	131,072	26	64
2 <sup>16</sup>	65,536	25	32
2 <sup>15</sup>	32,768	24	16
2 <sup>14</sup>	16,384	2 <sup>3</sup>	8
2 <sup>13</sup>	8,192	2 <sup>2</sup>	4
2 <sup>12</sup>	4,096	21	2
211	2,048	20	1

With respect to pl, the phase delays for the other data frequencies are then:

$$p^{2} = \frac{2^{4}-1}{2^{4}} p^{1} = \frac{15}{16} p^{1}$$

$$p^{3} = \frac{2^{8}+1}{2^{8}} p^{1} = \frac{257}{256} p^{1}$$

$$p^{4} = \frac{2^{12}-2^{4}-1}{2^{12}} = \frac{4079}{4096} p^{1}$$
(9-1)

The binary routine equivalent to the decimal one described in Sections 4, 5, and 6 is displayed in Figure 5. To minimize the number of shifting operations, the 13-bit phasemeter words are to extend from position 23 to 11. Thus the unit, equal to  $\frac{1}{8}$  foot, is located in position 11. All decimal numbers of this section refer to a unit in position 11.

We define all quantities with respect to pl as

P1 = p1 (9-2)  
P2 = 
$$p2-Q2 = \frac{15}{16}p1-Q2$$
  
P3 =  $p3-Q3 = \frac{257}{256}p1-Q3$   
P4 =  $p4-Q4 = \frac{4079}{4096}p1-Q4$ 

where the P's are the 13-bit binary phasemeter outputs which would exist at the time of the zero crossing of a reference phase (REF) generated within the DME. Since the read command, which is derived from range timing, is generally not coincident with the zero crossing of the reference phase, the actual phasemeter outputs, denoted by D's, equal the correspondent P's increased by the instantaneous reference phase at the time of reading. The reference phase is recorded as a positive 13-bit number compatible with the phasemeter outputs. Thus the P's are obtained from the D's as

Note that all quantities of Equation 9-3 are to appear as positive 13-bit numbers. Therefore, there should be no carry into the 14th bit. We now resolve the fine ambiguities;

$$S2 = 16P1-16P2 = 16p1-16(\frac{15}{16}p1-Q2)$$
 (9-4)

= p1+16Q2

Note that multiplication by powers of 2 may be programmed as shifting operations. S2 is a 17-bit binary number with four insignificant zeros as the last four bits. S2X equals P1 augmented by four binary X-bits at the beginning. The distinction between S2X and P1 is made, and the X-bits are shown in Figure 5, for the sole purpose of emphasizing that no operations are to be performed on these X digits. Yet, we refer to S2X as a 13-bit number, not counting the X-bits.

$$16Q2X = S2-S2X$$

(9-5)

16Q2X is also a 13-bit number augmented by four X-bits. In accordance with Section 4, the negative complement to 8192 is to be taken when 16Q2X exceeds half its range. In the decimal system, 10Q2X ranged from 0 to 999 and was to be complemented when exceeding 499. The binary 16Q2X ranges from 0 to 8191, or 0 to 1 111 111 111, and is to be complemented when exceeding 4095 or 0 111 111 111 111. Therefore complementation depends only on the first bit (position 23): if it is a one, then 16Q2 is the negative complement to 8192 of 16Q2X. Otherwise 16Q2 = 16Q2X.

For proper ambiguity resolution, the decimal 10Q2 may range from -500 to +499. Similarly, proper ambiguity resolution with the binary system will be obtained with 16Q2 between -4096 and +4095 or Q2 between -256 and +255. Having computed Q2,

$$S2C = S2 - 16Q2$$
 (9-6)

The coarse-ambiguity resolution follows closely Section 5.

S3 = 256 P3-256 P1 = 
$$256(\frac{257}{256}p1-Q3)-256p1$$
 (9-7)  
=  $p1-256Q3$ 

$$256Q3X = S3X-S3$$
 (9-8)

$$S3C = S3+256Q3$$
 (9-9)

S3 is a 21-bit number with eight insignificant zeros at the end. S3X, as shown in Figure 5, equals S2C augmented by four X-bits at the beginning. Again, the X-bits are to emphasize that no operations are to be performed on them. 256Q3X ranges from zero to 131071 just as the decimal equivalent 100Q3X ranged from zero to 9999. If it exceeds half its range, i.e. 65 535, the negative complement to 131072 is to be taken. Since numbers larger than 65 535 are identified by a one in position 27, negative complementation is to be performed when bit 27 is a one. For proper ambiguity resolution, 256Q3 may range from -65536 to +65535 or Q3 from -256 to +255.

The resolution of the very-coarse ambiguities follows Section 6. Since at this time, there is no particular use of obtaining S4 explicitely, we start with computing S40.

= p1+4096Q4

$$409604X = S40 - S4X \tag{9-11}$$

$$S4C = S40-4096Q4$$
 (9-12)

Since \$3C is a 21-bit number, so is \$40. \$4X equals \$3C augmented by four X-bits at the beginning, as shown in Figure 5. Again, the sole purpose of showing these X-bits is to indicate that no operations are to be performed on them. 4096Q4X appears as a number of 21 significant bits, and ranges from zero to 2 097 151. The negative complement to 2 097 152 will be taken when 4096Q4X exceeds half its range, i.e. 1 048 575, which is indicated by a one in position 31. For correct ambiguity resolution, 4096Q4 may range from -1048576 to +1048575 or Q4 from -256 to +255.

In the final averaging process

$$RA = S4C - \frac{1}{2}02 - \frac{1}{2}03 - \frac{1}{2}04 \tag{9-13}$$

where the last three bits of Q4 are omitted to avoid exceeding a word length of 36 bits. Probably more than just those 3 bits could be omitted from both Q3 and Q4, but the time saving does not appear significant and using the complete Q's eliminates round-off errors.

#### 10. THE IMPROVED METHOD OF AMBIGUITY RESOLUTION IN BINARY FORM

This routine is explained in detail for a decimal system in Section 8. See Figure 6 for a graphical aid. The resolution of the fine ambiguities is identical to that of Section 9:

$$S2 = 16P1-16P2$$
 (10-1)

$$1602X = S2-S2X (10-2)$$

$$S2C = S2-16Q2$$
 (10-3)

Now S2C is corrected by half the inconsistency of P1 and P2 as

$$R2C = S2C - \frac{1}{2}Q2$$
 (10-4)

R2C is a 22-bit binary number. Its last 18 bits, called P1C, are used as an improved reference for the resolution of the coarse ambiguities. We define now P3 with regard to this new reference as

$$P3 = \frac{257}{256} 1 - U3 \tag{10-5}$$

Then the coarse ambiguities are resolved as

$$S3A = 256P3 - 256P1C = 256(\frac{257}{256}P1C - U3) - 256p1C$$
 (10-6)

= p1C-256 U3

$$256U3X = S3X-S3A$$
 (10-7)

where S3X equals R2C with four X-bits added at the beginning. The negative complement is taken, as usual, when bit 27 is a one. Otherwise U3 = U3X.

$$S3C = S3A+256 U3$$
 (10-8)

Then we perform a second average

$$R3C = S3C - \frac{1}{3}U3$$
 (10-9)

We use the last 24 bits of R3C, called P1CC, as a reference for the resolution of the very-coarse ambiguities. Defining P4 with regard to this new reference

$$P4 = \frac{4079}{4096} p1CC - U4 \tag{10-10}$$

we compute S4A, U4, and S4C as

S4A = 4096P1CC-4096P4-16R3C

$$= 4096p1CC-4096(\frac{4079}{4096}p1CC-U4) - \frac{16}{4096}p1CC \qquad (10-11)$$

= p1CC+4096U4

$$4096U4 = S4A - S4X \tag{10-12}$$

where S4X equals R3C with four X-bits added at the beginning. After taking the negative complement of 4096U4X when bit 31 is a one,

$$S4C = S4A-4096U4$$
 (10.13)

Finally, the average range is

$$RA = S4C-\frac{1}{2}U4$$
 (10-14)

Figure 6 shows that the last bits of Q3 and Q4 are omitted when making the range corrections.

#### 11. SMOOTHING

Smoothing, or time-averaging, may be applied to either the phasemeter outputs P or to the final averaged range RA. The same precision will be attained in either case as long as the phasemeter noise is small enough as to not introduce ambiguities. For the case of large phasemeter noise, smoothing the raw phasemeter data would enhance the ambiguity resolution capability. Consideration of the merits of various smoothing techniques should be made after actual DME noise characteristics have been studied.

#### 12. DME TAPE FORMAT

DME data are recorded on standard low-density IBM tape in blocks of 120 36-bit words. One sample point, or sample, comprises 12 words, thus one block contains ten samples. The format of one sample is shown in Figure 7. Usually one 36-bit word contains two phasemeter words. Bits are numbered 0 to 35 according to the computer bit-weighting 2° to 2<sup>35</sup>. Word 9 identifies tape location with computer bit weighting in full. All numbers are positive, therefore no position is reserved for identifying a sign. Word sequence for one sample is as follows:

```
Word 1 bits 19 - 0 G-2 timing
```

- Word 2 Bits 35 18 reserved for identification code bits 17 - 5 DME phase-reference word
- Word 3 bits 35 23 D1 phasemeter cutput, Target 1 bit 22 reserved for additional quality indication bit 21 D1 phasemeter lock-on indication
  - bit 21 Dl phasemeter lock-on indication
  - bit 17 5 D2 phasemeter output, Target 1
  - bit 4 reserved for additional quality indication
  - bit 3 D2 phasemeter lock-on indication
- Word 4 bit 35 23 D3 phasemeter output, Target 1
  - bit 22 reserved for additional quality indication
  - bit 21 D3 phasemeter lock-on indication
  - bit 17 5 D4 phasemeter output, Target 1
  - bit 4 reserved for additional quality indication
  - bit 3 D4 phasemeter lock-on indication
- Word 5 Target 2, otherwise as Word 3
- Word 6 Target 2, otherwise as Word 4
- Word 7 Target 3, otherwise as Word 3
- Word 8 Target 3, otherwise as Word 4
- Word 9 12 Spare, reserved for Doppler data.

DME AMBIGUITY RESOLUTION WITH FOLLOWING RANGE CORRECTION

Figure 5

6 IMPROVED DME AMBIGUITY RESOLUTION WITH STEP-BY-STEP AVERAGING

DME TAPE FORMAT

### U. S. ARMY ELECTRONICS RESEARCH AND DEVELOPMENT ACTIVITY WHITE SANDS MISSILE RANGE NEW MEXICO

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Instrumentation Denartment, U. S. Army Electronics Research and Develowment Activity, White Sands Hissile Range, New Nextco DNE DATA REDUCTION, By D. T. Holberg, USA-ERDA-1, 29 rp incl illus, January 1963  UNCLASSIFIED Report  The ambiguity-resolution problem for a Distance Measuring Equipment (ONE), using four inderendent modulation frequencies in the 500-kc region, is described in mathematical terms. The method is develowed for a DNE model with decimal modulation-frequencies in the 400-kc region, is described in mathematical terms. The method is develowed for a DNE model with decimal modulation-frequencies are related by binary numbers. Finally, two digital-computer routines providing for both ambiguity resolution and accuracy increase by averaging over the four whase measurements are develowed.	Instrumentation Denartment, U. S. Army Electronics Research and Develonment Activity, White Sands Missile Range, New Mexico  DME DATA REDUCTION, By D. T. Holberg, USA-ERDA-1, 29 rp incl illus, January 1963  UNCLASSIFIED Report  The ambiguity-resolution problem for a Distance Measuring Equipment (DME), using four inderendent modulation frequencies in the 500-kc region, is described in mathematical terms. The method is developed for a DME model with decimal modulation-frequencies in the 500-kc region, is described in mathematical terms. The method is developed for a DME model with decimal modulation-frequencies are related by binary numbers. Finally, two digital-computer routines providing for both ambiguity resolution and accuracy increase by averaging over the four phase measurements are developed.

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